# Non-linear Transformation of the Economic System's Development

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# Concept of the linear dynamics model

- In economics, linear dynamic models are the primary tool for analyzing macroeconomic processes, forecasting financial markets, and modeling economic cycles. Their fundamental principles are:
- ***The principle of superposition***: If a system responds to an input signal $A_1$ with result $R_1$ and to signal $A_2$ with result $R_2$, then it will respond to the combined signal $(A_1 + A_2)$ with result $(R_1 + R_2)$. This is a fundamental property of linearity in dynamics.
- ***Linearity in parameters***: The model's prediction is calculated as a weighted sum of features plus an intercept term:

$$Y = \mu_0 + \mu_1 X_1 + \mu_2 X_2 + ... + \mu_n X_n$$

# Research objective

- However, economic factors are so strongly interconnected that coefficient estimates in linear equations become unstable even with the slightest change in empirical data. Furthermore, linear models are fundamentally incapable of predicting catastrophic system changes – specifically, the transition of the system to a new qualitative state.
- To effectively forecast changes in the state of an economic system (tracking phase trajectories), there is currently a shift toward using mathematical models of synergetic dynamics, which are fundamentally nonlinear in nature.
- This study examines the application of the nonlinear dynamics paradigm, using the analysis of an enterprise's financial condition as an example.

# The concept of nonlinear dynamics

- The synergetic economic paradigm considers the economy as an open, complex, non-linear and non-equilibrium system that is capable of self-organization.
- ***Synergetic effect*** (Emergency): the properties of the whole system as a whole exceed the sum of the properties of its individual parts.
- ***Nonlinearity***: a property in which the result of an action is not proportional to the action itself.

# Setting the task

- A qualitative analysis of the firm's financial and economic activity is presented for the single-product case.
- The model incorporates the minimum possible number of order parameters (economically significant dynamic variables); nevertheless, it demonstrates a wide range of potential development paths for the firm and identifies critical regimes (in an evolutionary sense).
- The financial state of the enterprise is described in continuous time, consequently, the mathematical model takes the form of a system of differential equations.

# Mathematical model

- The result of the analysis of financial and commodity flows is a system of two ordinary differential equations:

$$\begin{cases} \dfrac{dM}{dt} = Q_m p Y(L-Y) - K - \dfrac{Y}{\tau_2}, \\ \dfrac{dY}{dt} = n\dfrac{M}{\tau_1} - Q_m p Y(L-Y). \end{cases}$$

Here, $M$ represents the volume of monetary working capital;

$Y$ is the quantity of finished goods at market prices;

$K$ is capital investments from own funds;

$\tau_1$ is the turnover time;

$1/\tau_2$ is the share of working capital spent on storing a unit of finished goods per unit of time;

$p$ is the market price per unit of product;

$0.5L$ is the volume of product corresponding to maximum demand;

$n = p/p_0$ , where $p_0$ is the cost price.

$r = \frac{n}{n-1} > 0$

# Singular points

By solving the system of differential equations, we find its singular solutions, which correspond to the static equilibrium positions of the nonlinear dynamic system under study:

$$\begin{cases} M^* = \frac{1}{n} Y^* (1 - Y^*), \\ \left(Y^*\right)^2 - \left(1 - \varepsilon r\right) Y^* + br = 0, \end{cases}$$

where the parameters are:

$$\varepsilon = \frac{1}{Q_m L^2 p \tau_2}, \qquad b = \frac{K}{Q_m L^2 p},$$

and $r = \frac{n}{n-1} > 0$, since $n > 1$.

# Existence of positive solutions

- The algebraic equation in $Y^*$ has positive solutions if the following conditions are met:

$$\begin{cases} 1-\varepsilon r > 0, \\ (1-\varepsilon r)^2 \geq 4br. \end{cases}$$

- If these conditions are met, the solutions themselves take the form:

$$Y_{1,2}^* = \frac{1-\varepsilon r}{2} \pm \sqrt{\frac{(1-\varepsilon r)^2 - 4br}{4}}$$

For the sake of certainty, we will assume that $Y_2^* > Y_1^* > 0$.

# Stability analysis of singular points

Let us consider the behavior of the system in the neighborhood of the singular point $\left(M^*, Y^*\right)$ using the new variables $m = M - M^*$ and $y = Y - Y^*$.

In terms of these variables, we obtain a system of differential equations:

$$\begin{cases} \dfrac{dm}{dt} = -m + \left(1 - \varepsilon - 2Y^*\right) \cdot y - y^2, \\ \dfrac{dy}{dt} = \dfrac{\alpha \cdot r}{r-1} m - \alpha \cdot \left(1 - 2Y^*\right) \cdot y + \alpha \cdot y^2, \end{cases}$$

where

$$\alpha = Q_m L^2 p \tau_1$$

The matrix of the linear part corresponding to this system is:

$$F = \begin{pmatrix} -1 & 1 - \varepsilon - 2Y^* \\ \dfrac{\alpha \cdot r}{r-1} & -\alpha \cdot \left(1 - 2Y^*\right) \end{pmatrix}$$

# Stability analysis of singular points

- Analysis shows that point $Y_1^*$ corresponds to ***a saddle-type*** equilibrium position. That is, the equilibrium state at this point is unstable, and phase-plane trajectories approach the point along certain directions while moving away from it along others.

- The singular point $Y_2^*$ is ***a stable node*** (***or focus***). This is a type of stationary point in a dynamical system where the equilibrium state is asymptotically stable. This means that, following any small perturbation, the system eventually returns to this point on its own.

# The emergence of bifurcations

- However, there are two critical values characterizing a firm's total costs:
- $$b_1^* = \frac{(1-\varepsilon r)^2}{4r}, \quad b_2^* = \frac{1+\alpha}{\alpha} \cdot \frac{1-2\varepsilon r}{4r} \quad \text{and} \quad b_1^* > b_2^*$$
- If costs exceed this value $b > b_1^*$, that is, if the warehouse is overstocked, the two points merge and a loss of stability occurs, accompanied by ***a saddle-node bifurcation***. Such a bifurcation models situations of sudden structural shifts: market crashes, the instantaneous disappearance of demand, or a system's transition into a protracted crisis amidst gradual changes in external factors.
- As the value of $b_2^*$ approaches point $Y_2^*$, which corresponds to the state of stable equilibrium, a transition occurs from a stable focus to an unstable one. ***A limit-cycle birth bifurcation*** takes place, resulting in self-sustained oscillations.

# Conclusions

- In conclusion, it is worth re-emphasizing that the key feature of the proposed model is its nonlinearity. This gives rise to system properties such as the existence (in this case) of two equilibrium positions and the occurrence of bifurcation upon the smooth variation of certain parameters (specifically, the level of expenditure).

- The proposed mathematical model can serve as a basis for simulation modeling to illustrate the system's behavior near points of stability loss.

# Thank you for your attention

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