DOCTORAL RESEARCH

Determining the Economically Justified Recycled-Content Ratio in Manufacturing

A process-capability and quality-cost decision model

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Regulation now sets the target

FROM 2030

30%

Recycled content in contact-sensitive PET packaging

FROM 2030

35%

Recycled content in other plastic packaging

FROM 2040

65%

Rising obligation on the same category

Comparable obligations are arriving for end-of-life vehicles. The direction is settled; the quantities are legally binding.

Regulation (EU) 2025/40 · figures to be confirmed against the Official Journal text

Regulation does not say how much a process can absorb

WHAT FIRMS DO NOW

- Confine recyclate to non-critical parts
- Cap it at a percentage fixed by habit
- Run blend trials on two or three lots
- Check short-term process capability

WHAT IS MISSING

No method connects the measured variability of the secondary material, through process capability, to the defect rate and its cost and so to a limit anyone can defend.

The limit is set by heuristic, not by analysis

Why secondary material is different

Virgin feedstock

Made to specification from controlled inputs. Lot-to-lot variation is small next to the variation within a lot.

Post-consumer recyclate

Recovered from a heterogeneous waste stream. Lot-to-lot variation can be the **dominant** component of total variation.

Short-term capability indices are built from variation *within* a subgroup. They exclude the between-lot component by construction.

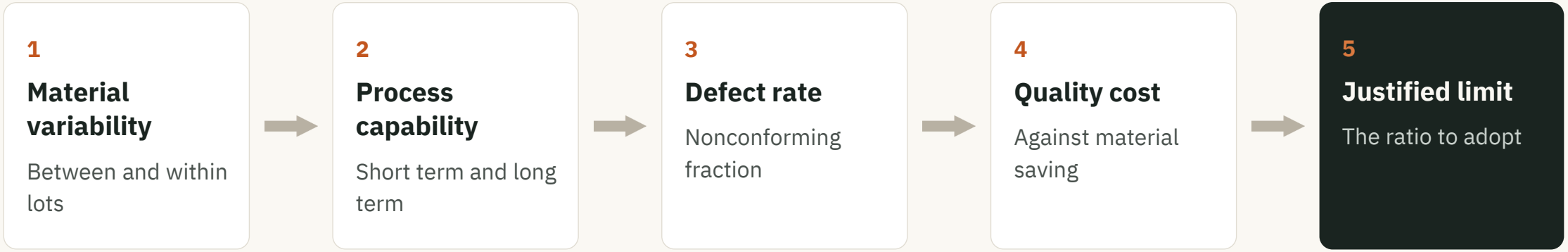
The qualification method omits exactly the component recyclate contributes most

THE RESEARCH QUESTION

How much secondary material can a process absorb before the quality cost outweighs the material saving and how would a firm know?

Unit of analysis: the process–material pair, not the firm

The model is one chain



Each link exists in the literature. The chain does not. Joining it is what converts a heuristic into a calculation.

Capability is not a fixed property here

The gap between long-term and short-term variance, at a substitution ratio *r*:

$$\sigma LT^2(r) - \sigma ST^2(r) = \beta_1{}^2 \cdot \mathbf{r^2} \cdot \sigma B^2$$

The gap is **quadratic in the decision variable**. It is not a fixed offset to measure once.

⚠ A gap measured during a trial at 10% recyclate understates the gap at the 40% eventually adopted — by a factor of sixteen.

Qualify low, run high, and the error grows with the square of the ratio

RESULT 1

The bias has a closed form

Short-term qualification inflates the substitution limit by exactly

$$1 + \kappa$$

$$\kappa = \sigma B^2 / (\delta^2 + \sigma v^2 + \sigma W^2)$$

σB² between-lot variance of the recyclate

δ mean property shift from virgin

σv², σW² virgin and within-lot variance

A single dimensionless parameter decides whether the error matters

RESULT 2

And it is bounded

$$\kappa \le \sigma B^2 / (\sigma v^2 + \sigma W^2)$$

The mean shift drops out of the ceiling entirely.

Why this matters

The error does not run away as recyclate approaches parity with virgin material. It saturates at the recyclate’s own between-to-within variance ratio a number a plant can estimate from its goods-in records.

A worst case that can be computed before any trial is run is worth more to a quality manager than a point estimate that cannot.

RESULT 3

A second bias, from estimating

The limit is convex in the variance being estimated. So even an unbiased estimate of σ_B^2 gives an **upward** biased limit stacking on the first error.

Lots qualified	κ = 0.10	κ = 0.30	κ = 0.50	κ = 1.00
3 lots	0.8%	5.4%	11.2%	25.2%
5 lots	0.4%	2.7%	5.6%	12.6%
10 lots	0.2%	1.2%	2.5%	5.6%
20 lots	0.1%	0.6%	1.2%	2.7%

More specimens per lot does **not** help. The variance is identified by the number of lots, not the number of tests within them.

Estimation bias in the substitution limit · ten specimens per lot

RESULT 4

The usual check does not catch it

Hold the short-term capability index at the conventional 1.33 acceptance gate. The true long-run defect rate behind it:

Process noise σp/σB	0.25	0.5	1.0	4.0
κ = 0.10	17 145	1 227	128	37
κ = 0.30	29 893	2 501	196	38
κ = 0.40	37 118	3 436	244	39

⚠ The gate implies 32 ppm. At the left-hand column the process passes while running at more than a thousand times that rate.

Defect rate in ppm · the better the process control, the worse the blind spot

THE FINDING

The conditions that make substitution look safe are the conditions under which the standard method fails.

Small mean penalty

Nothing in a trial prompts caution, yet κ is at its largest

Tight process control

The capability index is cleanest, and hides the lot effect best

Where the condition does not hold

Commodity polypropylene

Recyclate modulus falls about 27% below virgin. The mean shift dominates.

$$\kappa \approx 0.02$$

Bias 2.2%. Existing practice is adequate.

Why report this

It is the system with the best published data, and the one that motivated the work.

A condition satisfied everywhere would deserve suspicion. The negative case is what makes the positive ones credible.

The contribution is a condition, not a claim that every manufacturer has a problem.

Three changes to qualification practice

1

Compute κ before qualifying

From the mean shift and the variance ratio in goods-in records. Below 0.05, carry on as you are.

2

Decompose the virgin baseline too

The direction of the error depends on virgin and recyclate between-lot variance together — not the recyclate alone.

3

Qualify over lots, not specimens

Where κ is material, ten lots or more. Extra tests inside a lot buy nothing.

None of these requires new instrumentation or new data collection

The wider programme

STUDY ONE

How firms decide today

Specification documents, inspection procedures and non-conformity records across several plants

STUDY TWO · PRESENTED TODAY

The decision model

Analytical derivation and simulation; the materiality condition and its bound

STUDY THREE

Validation on plant data

Back-testing predictions against outcomes at known recycled-content levels

The open empirical question: nobody has measured between-lot variance components for real secondary-material streams.

IN ONE SENTENCE

Qualifying a recyclate on two or three lots overstates how much a process can absorb by an amount one computable number decides, and conventional capability checks will not reveal.

Compute κ from the records you already hold

Qualify over lots, not specimens

Decompose the virgin baseline as well

Thank you — questions welcome